REVIEW

Konforme Abbildung. By ALBERT BETZ. 2nd revised edition. Springer, 1964. 407 pp. DM 49.60 or £4 12*s*.

This book stems from the author's conviction that, for engineering students and others interested primarily in applications, the properties of analytic functions of a complex variable follow more naturally from the geometry of conformal maps than from the conventional approach in which conformal mapping is treated as a branch of complex-variable theory. Accordingly, the first 150 pages are devoted largely to constructing various elementary transformations either from the principle of conservation of shape of infinitesimal surface elements, or from the current lines and equipotentials of simple electrostatic fields; and to examples from steady heat conduction, hydrodynamics, and so on. The Laplace equation appears first on p. 85, and complex variables only on p. 156.

This beginning strikes me as misguided. The pictorial or geometrical aspects of the subject are, of course, vital; but they are so intimately related to the idea of a derivative

$$\frac{dw}{dz} \equiv \lim_{|\delta z| \to 0} \frac{w(z+\delta z)-w(z)}{\delta z}$$

independent of the direction of δz (so that under the associated mapping all line elements at an ordinary point are magnified by the same amount and rotated by the same amount), and manipulation of $w(z)$ is usually so much easier than that of its real and imaginary parts, that to delay the use of complex variables makes the student's task harder than it need be. An approach combining graphical and analytical ideas from the first, and emphasizing the remarkable properties of analytic functions by contrasting them (as Prof. Betz does not) with non-differentiable functions of z (like $|z|^2$), is well within the grasp of the modern engineering student, who has little time to spare for primitive methods that must ultimately be replaced by more efficient ones.

The second half of the book, in which analytic functions are used freely, proceeds to increasingly advanced transformations like the Schwarz-Christoffel mapping, that of a circular triangle on to a half-plane, and those associated with Jacobian and Weierstrassian elliptic functions. The applications are mainly to hydrodynamics, and there is a chapter on free-streamline flows with emphasis on cascades. The author aims not at completeness and rigour, but at simplicity and clarity, and in this he is largely successful. (A glaring exception is his use of the same letter for the upper limit and for the variable of integration of many definite integrals; an unjustifiable, if common, practice which causes considerable confusion in his treatment of elliptic integrals.)

Comments on the *text* of this book are, however, of secondary importance, for its appeal lies, and is intended to lie, in the 268 splendid figures, for which the text serves mainly as a commentary. The displayed streamline patterns of a large number of complex potentials give a wealth of detailed information without ever appearing vulgar or cluttered, and illustrate vividly and beautifully the analytical properties of the many functions considered. Masters of classical analysis may feel no need for these pictures, but most readers will surely welcome them both as an aid to understanding analytic functions of a complex variable, and as a source of delight in themselves.

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